Title : **LEGENDRE SPLINE COLLOCATION METHOD TO SOLVE INTEGRO DIFFERENTIAL EQUATIONS AND ITS APPLICATIONS**

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SYNOPSIS

The present thesis entitled "**Legendre Spline Collocation method to solve Integro differential equations and its applications** " submitted for the award of Ph.D. degree is based on the study of relevant fields on collocation methods and Integro Differential Equations arising in biology, computer graphics and Engineering with programming in MATLAB.

Most mathematical models used in the natural sciences and engineering are based on ordinary differential equations, partial differential equations, integral equations and integro differential equations.

As a matter of fact very few integral equations and integro differential equation have closed form solutions and even if such solutions exist, except for a very simple case, lots of mathematical exercises are required to obtain these solutions. However, it is still desirable to find these closed form solutions so that maximum information can be extracted from these solutions. In the case where analytic solution of an integro differential equation is not possible or rather too difficult to obtain, one looks for approximation to it as a compromise between accuracy and labor involved in getting closed form solution. This is where numerical methods prove to be crucial.

Integro-differential equation is an equation that contains both derivatives and integrals of the unknown function. Systematic study of integral equations and integro differential equation started from the works of Volterra, where he transformed an integral equation

$$\int_0^t \kappa(t,s)y(s)ds = g(t), t \in [0,T]$$

by differentiation with respect to t, into an intergal equation of the form

$$y(t) = \int_0^t \kappa(t,s)y(s)ds + f(t), t \in [0,T]$$

which is latter known as Volterra integral equation of the second kind and Fredholm, where he gave necessary and sufficient conditions for solvability of integral equations later known as Fredholm integral equations of the second kind which is of the form

$$y(t) = \int_0^b \kappa(t,s)y(s)ds + f(t), t \in [0,b]$$

Overview

The aim of this research work is to develop the new accurate numerical methods named Legendre spline, Legendre B-spline, Bernstein polynomial to solve linear and nonlinear integro differential equations of different order with given initial and boundary conditions. Also we have explore some applications of the integro differential equations to the problems arising in biology.

A standard method for solving integro-differential equations is the collocation method, where one looks for an appropriate solution in a finite dimensional space of candidate solutions (usually, polynomials up to a certain degree) and a number of points in the domain (called *collocation points*), and to select that solution which satisfies the given equation at the collocation points. It is known that B-splines can resolve complex curves with drastically fewer data than can their standard shape function counterparts. B-spline collocation method has the attractive features like: (1) Any level of accuracy can be achieved by using B-splines of sufficient degree with sufficient numbers of splines segments. (2) There are no truncation errors associated with the method; the errors occur in how closely the B-spline curves can represent a solution variable.(3) A computational mesh in the standard sense is not required; the

knot meshes that are required are expected to be generated during the solution procedure.

The present research work focuses on some of the numerical methods discussed above. The results from the above work would enable its further applications in obtaining more accurate solutions of several complex physical problems relative to various branches of science.

Generally it is very difficult to solve integro differential equation analytically. So we have developed an efficient numerical method named natural Legendre spline collocation and Legendre B-spline collocation method of various degree for solving first order and higher order integro differential equations. Also we have Applied Bernstein polynomial of various degree using collocation method for solving first order and higher order integro differential equation. As an applications of the above mentioned collocation methods we have study the Integro-differential equations related to immunological problems.

This thesis consist seven chapters.

Chapter 1

First chapter is an introductory chapter where we discussed some formal mathematical concepts required to develop highly accurate numerical schemes for the solution of integro differential equations. Also few of the previous research work on solving integro differential equations using various numerical methods are studied. This chapter also presents a summary of the work done in the thesis.

Chapter 2

In chapter 2, we presented a basic concept of Bernstein polynomial and its properties with its importance for solving integro differential equation.

Numerical results were given to confirm the theoretical analysis and the results compared with those obtained by known methods to demonstrate the utility of our method. The method is successfully applied to both linear and nonlinear problems. And the related work is published in the International Journal of Applied Mathematical Sciences, Vol. 5, 2011, Hikari publication.

Chapter 3

Chapter 3contains basics of Legendre polynomial and its properties with its importance for solving integro differential equations. Here derivation of natural Legendre Spline method has been discussed in detail. We have discussed different order of natural Legendre Spline method such as Cubic Legendre Spline, Quartic Legendre Spline, Quintic Legendre Spline and septic Legendre spline. Numerical results were given to confirm the theoretical analysis and the results compared with those obtained by known methods to demonstrate the utility of our method. The method is successfully applied to both linear and nonlinear problems.

Chapter 4

In chapter 4, we presented a basic concept of B-spline and discussed the difference between B-spline and natural spline. Here we have developed approximate methods to solve integro differential equation by using cubic Legendre B- spline. Also we have discussed quartic Legendre B-spline, quintic Legendre B-spline and septic Legendre B-spline. The developed methods are calibrated by considering various Integro differential equations. Numerical results gives the confirmation of the theoretical analysis of the method. Also the results are compared with those obtained by known methods shows the applicability and reliability of the methods.

Chapter 5

In chapter 5, we have applied Bernstein polynomial method, Legendre spline and Legendre B-spline collocation method to solve system of linear and nonlinear integro differential equations of various order. The comparison of Numerical results with those obtained by known methods demonstrate the utility of our method.

Chapter 6

Most of the real world problems are three dimensional so to get solutions of such problems we have to extend our method to solve partial integro differential equations. In this chapter we have used Bernstein polynomial and cubic Legendre B-spline to solve partial integro differential equations. The results are compared with the results available in the literature which proves the accuracy of the present methods.

Chapter 7

Integro differential equations have wide number of applications in various fields like computer graphics, ecology, population dynamics, biology, medical sciences and in engineering. Here we have studied biological application of immunological problems. We have applied our methods with computer simulations for analysis of the acquired immune response to viral infection. The considered mathematical model is the system of partial integro differential equations. We have conclude that mathematical models of immunological phenomena may be useful for better understanding of processes occurring in living organisms. Also we have applied numerical methods developed in chapter 3 and chapter 4 to solve the convection-diffusion integro differential equations. These equations occur in many applications such as in the transport of air and ground water pollutants, oil reservoir flow etc.

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